Probabilistic Load Flow Based on Generalized Polynomial Chaos

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***Abstract*—An analytical method based on generalized polynomial chaos (gPC) is proposed for probabilistic load flow (PLF). The method preserves the nonlinearity of power flow equations whose rectangular formulations are adopted to facilitate the gPC expansion. The feasibility of the method is demonstrated by case studies from a 9-bus system.**

***Index Terms*—Probabilistic load flow, generalized polynomial chaos, orthogonal polynomials, Galerkin method.**

I. Introduction

With the rapid increase of wind and solar generation, power system operations are challenged by greater variabilities and uncertainties. Because the PLF can provide comprehensive information about power system operation with random factors, it attracts more attention recently.

PLF methods could be roughly divided into two categories: simulation methods and analytical methods [1]. Monte Carlo simulation (MCs) is one of the most famous simulation methods, which is straightforward and accurate but time consuming. Analytical methods, on the other hand, reduce computation burden with the help of mathematic analysis typically linearization, but compromise the accuracy.

The gPC method first appeared in 2002 as an extension of the polynomial chaos method [2], and shortly has become a well adopted method for stochastic analysis of complex systems, such as fluid dynamic and finite element analysis [3]. This letter gives a brief introduction to the gPC method in the context of single continuous random variable, and then explores the PLF problem with multiple random variables.

II. Generalized Polynomial Chaos Method

Orthogonal polynomial theory is the base of the gPC method. A monic polynomial $Q_n(x)$ is defined as $\sum_{i=0}^{n} a_i x^i$, where the nonnegative integer n is the degree of the polynomail and $a_n = 1$. A system of polynomials $\{Q_n(x)\}$ is an orthogonal system of polynomials with respect to some weight $\omega(x)$, if the following orthogonality relations hold [3]

$$\int_S Q_m(x)Q_n(x)\omega(x)dx = \gamma_n \delta_{mn}, \tag{1}$$

where S is the support of variable x, γ_n is a nonzero constant, and δ_{mn} is the Kronecker delta function, where $\delta_{mn} = 0$ if $m \neq n$ and $\delta_{mn} = 1$ if $m = n$.

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TABLE I
Correspondence Between the Types of Generalized Polynomial Chaos and Their Underlying Random Variables

Distribution of Z	gPC basis polynomials	Support
Gaussian	Hermite	$(-\infty, \infty)$
Gamma	Laguerre	$[0, \infty)$
Beta	Jacobi	$[a, b]$
Uniform	Legendre	$[a, b]$

The gPC method adopts the orthogonal polynomials as the basis functions of function expansion and interprets the weight as probability density function (PDF), so that a random function can be approximated by the linear combination of orthogonal polynomials of random variables.

Let $f(Z)$ be a scalar function of a random variable Z, then the N-order gPC expansion of $f(Z)$ is defined as

$$f(Z) \approx f_N(Z) = \sum_{i=0}^{N} f_i \Phi_i(Z). \tag{2}$$

$\Phi_i(Z)$ is the orthogonal polynomial basis function and satisfies

$$E[\Phi_i(Z)\Phi_j(Z)] = \gamma_i \delta_{ij}, \tag{3}$$

where γ_i is a constant, E is expectation operator, defined as $E[g(Z)] = \int_S g(z)\rho(z)dz$, $g(Z)$ is a continuous function, $\rho(Z)$ is the PDF of Z.

With (2) and (3), the gPC expansion coefficient f_i can be derived as follows.

$$f_i = \frac{1}{\gamma_i} E[f(Z)\Phi_i(Z)]. \tag{4}$$

The essential idea behind (2) is that $f(Z)$ is approximated in an $(N+1)$-dimensional space spanned by gPC bases. If $f(Z)$ is square integrable with respect to $\rho(z)$, f_N converges to zero exponentially when N goes to infinity [3].

Once we have the gPC expansion of $f(Z)$ as shown in (2), its expectation and variance can be expressed directly by the gPC expansion coefficients, i.e., f_0 and $\sum_{i=1}^{N} \gamma_i f_i^2$ respectively, and the PDF of $f(Z)$ can be approximately obtained based on (2) and the MCs sampled values of Z [3].

The gPC method uses 4 types of orthogonal polynomials from Askey scheme for describing continuous random variables. Table I shows the correspondence correlations between the types of gPC and their underlying random variables. If the PDF of Z does not fall into these 4 types, a transformation should be taken [3].

Consider the following stochastic nonlinear scalar equation

$$g(x, p(Z)) = 0, \tag{5}$$

where x is an unknown random variable, Z is an independent random variable with known distribution, and $p(Z)$ is a known input function. With a properly chosen basis $\Phi_k(Z)$, the N-order gPC expansions of x and $p(Z)$ can be written as

$$x(Z) \approx \sum_{k=0}^{N} x_k \Phi_k(Z) \quad \text{and} \quad p(Z) \approx \sum_{k=0}^{N} p_k \Phi_k(Z), \qquad (6)$$

where x_k and p_k are the expansion coefficients. Because $p(Z)$ is known, p_k can be directly obtained by using (4).

By substituting (6) into (5) and projecting the equation onto each basis $\Phi_m(Z)$ $(m = 0, 1, 2, \ldots, N)$, we have

$$E[g(\sum_{k=0}^{N} x_k \Phi_k(Z), \sum_{k=0}^{N} p_k \Phi_k(Z))\Phi_m(Z)] = 0. \qquad (7)$$

After evaluation of the expectation, Z disappears and $N+1$ deterministic equations are formed. Meanwhile, the unknown random variable x has been transformed into $N+1$ unknown coefficients x_k. Any conventional methods, such as Newton-Raphson method, can be used to solve the equations for x_k.

For the situation with multiple random variables and vector functions, the above principle remains unchanged [3]. However, after expansion, the dimension of deterministic equations and unknown coefficients will be $N_{gpc} = \frac{(N+p)!}{N!p!}$ times larger, where N is the order of gPC expansion and p is the number of independent random input variables. Therefore, gPC method would suffer the curse of dimensionality in complex cases.

III. Probabilistic Load Flow Using gPC Method

In order to facilitate the calculation of (7) for PLF, the power flow equations in rectangular form are adopted, because they are quadratic and hence can be easily expanded without any truncation error higher than 2 order. Following the traditional symbols, we have the following deterministic nonlinear equations to approximate the original PLF.

$$\sum_{j=1}^{N_{bus}} \sum_{k=0}^{N_{gpc}} \sum_{l=0}^{N_{gpc}} [G_{ij}(e_i^k e_j^l + f_i^k f_j^l) + B_{ij}(-e_i^k f_j^l + f_i^k e_j^l)] \cdot E[\Phi_k \Phi_l \Phi_m] - \sum_{k=0}^{N_{gpc}} P_i^k E[\Phi_k \Phi_m] = 0, \qquad (8)$$

$$\sum_{j=1}^{N_{bus}} \sum_{k=0}^{N_{gpc}} \sum_{l=0}^{N_{gpc}} [G_{ij}(f_i^k e_j^l - e_i^k f_j^l) - B_{ij}(e_i^k e_j^l + f_i^k f_j^l)] \cdot E[\Phi_k \Phi_l \Phi_m] - \sum_{k=0}^{N_{gpc}} Q_i^k E[\Phi_k \Phi_m] = 0, \qquad (9)$$

$$\sum_{k=0}^{N_{gpc}} \sum_{l=0}^{N_{gpc}} (e_i^k e_i^l + f_i^k f_i^l - V_i^k V_i^l) E[\Phi_k \Phi_l \Phi_m] = 0, \qquad (10)$$

where $\Phi_k(Z)$ and $\Phi_l(Z)$ are the orthogonal polynomial basis, $\Phi_m(Z)$ is the project basis, symbols such as e_i^k and f_i^k are the expansion coefficients for the real and imaginary parts of bus voltages, symbols such as V_i^k, P_i^k and Q_i^k are the expansion coefficients for bus voltage magnitude and power injections.

Coefficients V_i^k, P_i^k and Q_i^k can be worked out directly using (4) once gPC bases are set up, because V_i, P_i and Q_i are known in PLF. Coefficients e_i^k and f_i^k should be derived from (8)–(10). Thereafter the statistical quantities and PDFs of system variables can be obtained.

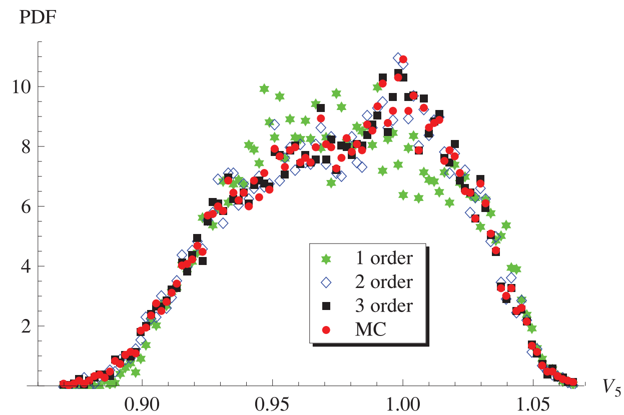

Fig. 1. The PDFs of V_5 obtained by MCs and gPC method.

IV. Numerical Examples

To demonstrate the feasibility of the proposed method, a well-known 9-bus system from [4] is adopted. Here we assume the load of bus 5 varies between 0 and 200% uniformly, the voltage of bus 3 fluctuates around its initial value normally with a standard variance of 0.03624, the real power output of bus 2 varies between 0 and 200% following Beta distribution with the shape parameters $\alpha = 4$ and $\beta = 2$. Other parameters are kept unchanged. Raw MCs with 10^4 samples serves as the benchmark for comparison.

Fig. 1 shows the PDFs of V_5 obtained by MCs and the proposed method with 1-, 2- and 3-order of gPC expansion. It can be seen that the PDF from 3-order expansion matches the MCs results quite well, while that from 1-order expansion suffers a large deviation. The expectation and variance percent error of V_5 from 2-order expansion are 0.000595% and -0.0815% respectively, which are the largest absolute percent errors among all bus voltage magnitudes. This acceptable result indicates that the nonlinearity of this 9-bus stressed system can be well embraced by a 2-order gPC method.

V. Conclusion

The gPC-based PLF method is a new powerful method that transforms the original PLF equations into a set of high-dimension deterministic equations and hence can have good computation performance, well handles the system nonlinearity and hence can achieve good computation accuracy.

However, in practical application, this prototypical method may suffer the curse of dimensionality if a higher order gPC expansion is adopted or many independent random variables exist. How to solve this type of problems is our future work.

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